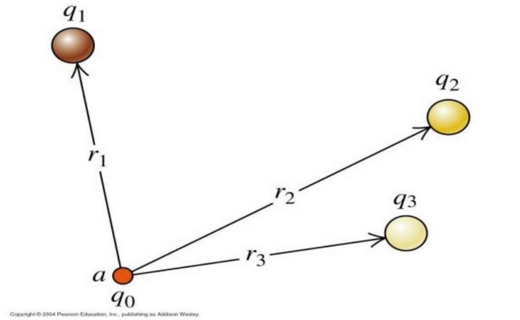
Electric Potential

Recall Case 1 from before



The potential energy of the test charge, q_0, was given by

$$PE_{q_0} = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_0 q_i}{r_i}$$

Notice that there is a part of this equation that would remain the same regardless of the test charge, q_0, placed at point *a*

The value of the test charge can be pulled out from the summation

$$PE_{q_0} = q_0 \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i}$$

Electric Potential

We define the term to the right of the summation as the electric potential at point *a*

$$\boldsymbol{Potential_a} = \sum_{\boldsymbol{i}} \frac{1}{4\pi\varepsilon_0} \frac{\boldsymbol{q_i}}{\boldsymbol{r_i}}$$

Like energy, potential is a *scalar*

We define the potential of a given point charge as being

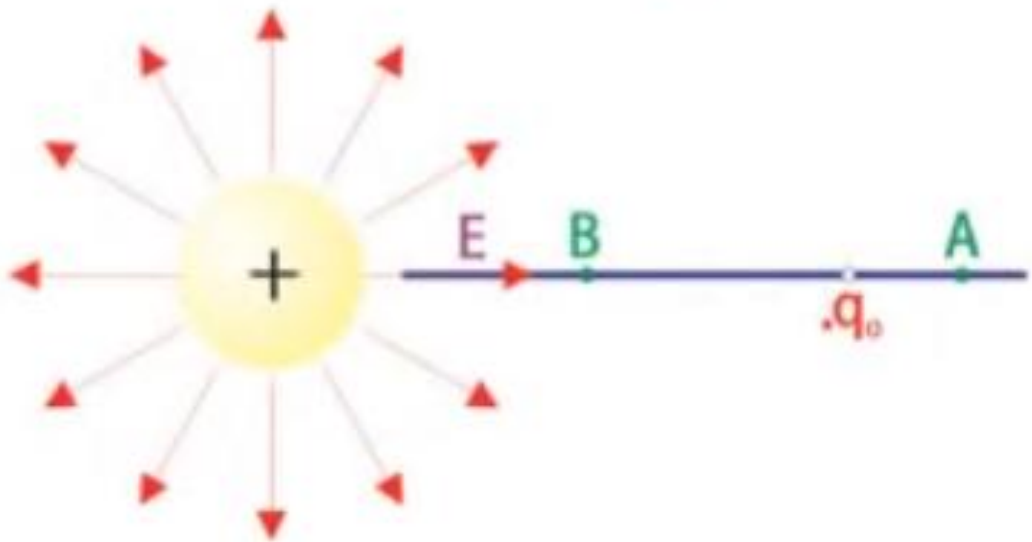
$$\boldsymbol{Potential} = \boldsymbol{V} = \frac{1}{4\pi\varepsilon_0} \frac{\boldsymbol{q}}{\boldsymbol{r}}$$

This equation has the convention that the potential is zero at infinite distance

Definition of Electric Potential Difference

We define the potential difference between two points *A* **and** *B* **as** the work done by an external agent in moving a test charge q_o from *A* to *B* *i.e.*

$$V_B - V_A = \frac{W_{AB}}{q_o}$$



The unit of the potential difference is (*Joule/Coulomb*) which is known as *Volt (V)*

Electric Potential

The potential at a given point

Represents the potential energy that a positive unit charge would have, if it were placed at that point

It has units of

$$\textbf{Volts} = \frac{\textbf{Energy}}{\textbf{charge}} = \frac{\boldsymbol{joules}}{\boldsymbol{coulomb}}$$

Electric Potential

General Points for either positive or negative charges

The Potential *increases* if you move in the direction *opposite* to the electric field

and

The Potential *decreases* if you move in the *same* direction as the electric field

Work and Potential

The work done by the electric force in moving a test charge from point *a* to point *b* is given by

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0\vec{E}\cdot d\vec{l}$$

Dividing through by the test charge q_0 we have

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

Rearranging so the order of the subscripts is the same on both sides

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}$$

Potential

From this last result $$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$$

We get $$dV = -\vec{E} \cdot d\vec{l} \text{ or } \frac{dV}{dx} = -E$$

We see that the electric field points in the direction of *decreasing* potential

We are often more interested in potential differences as this relates directly to the work done in moving a charge from one point to another

Units for Energy

There is an additional unit that is used for energy in addition to that of joules

A particle having the charge of *e* (1.6×10^{-19} C) that is moved through a potential difference of 1 Volt has an increase in energy that is given by

$$\boldsymbol{W} = \boldsymbol{qV} = 1.6 \times 10^{-19} \, \boldsymbol{joules}$$
$$= 1 \, \boldsymbol{eV}$$

Potential Gradient

The equation that relates the derivative of the potential to the electric field that we had before

$$\frac{dV}{dx} = -E$$

can be expanded into three dimensions

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{E} = -\left(\hat{i}\frac{dV}{dx} + \hat{j}\frac{dV}{dy} + \hat{k}\frac{dV}{dz} \right)$$

Potential Gradient

- For the gradient operator, use the one that is appropriate to the coordinate system that is being used.

Example

The electric potential in a region of space is given by

$$V(x) = 3x^2 - x^3$$

The x-component of the electric field E_x at $x = 2$ is

$\boxed{\text{(a)} \quad E_x = 0}$ (b) $E_x > 0$ (c) $E_x < 0$

We know *V(x)* "everywhere"

To obtain E_x "everywhere", use

$$\vec{E} = -\vec{\nabla} V \Rightarrow E_x = -\frac{dV}{dx} \Rightarrow E_x = -6x + 3x^2$$

$$E_x(2) = -6(2) + 3(2)^2 = 0$$

Example 2

The electric potential in a region of space is given by

$$V(x,y,z) = 5x^3 - 3y^2 + 6z^4$$

Find the electric field in this space at x=2, y=3, z=4

We know *V(x,y,z)* "everywhere"

To obtain $E_{x,y,z}$ "everywhere", use

$$\vec{E} = -\vec{\nabla}V \Rightarrow E_x = -\frac{dV}{dx} \Rightarrow E_x = -15x^2$$

E_x=-15(4)=-60

$$E_y = -\frac{dV}{dy} \;\Rightarrow\; E_y = 6\,y \;\Rightarrow\; E_y = 18$$

$$E_z = -\frac{dV}{dz} \;\Rightarrow\; E_z = -24\,z \;\Rightarrow\; E_z = -96$$